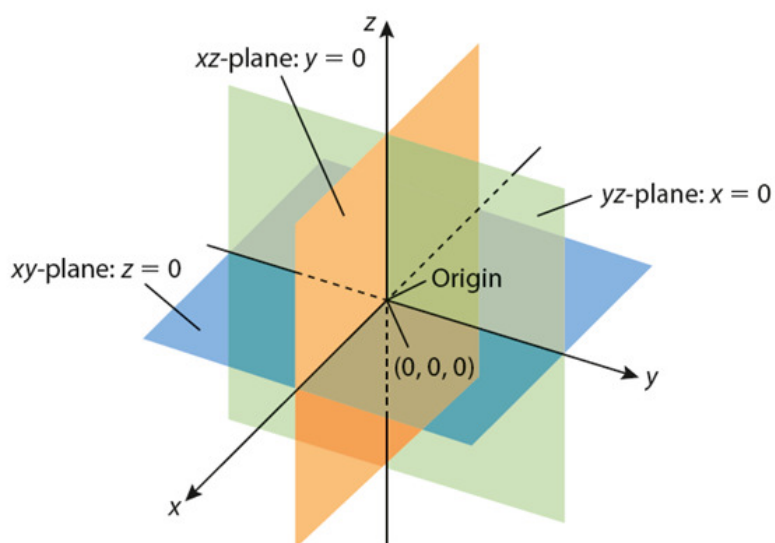
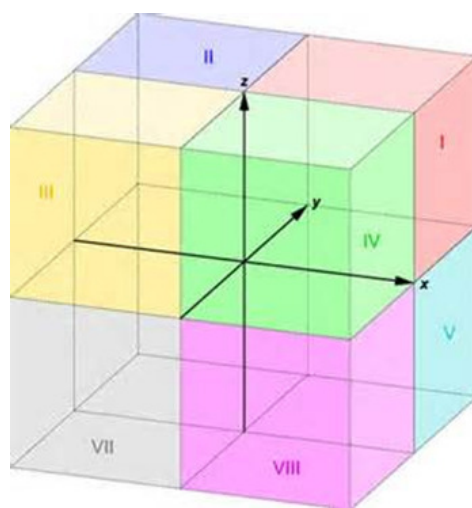


14-1 Vectors in 3D

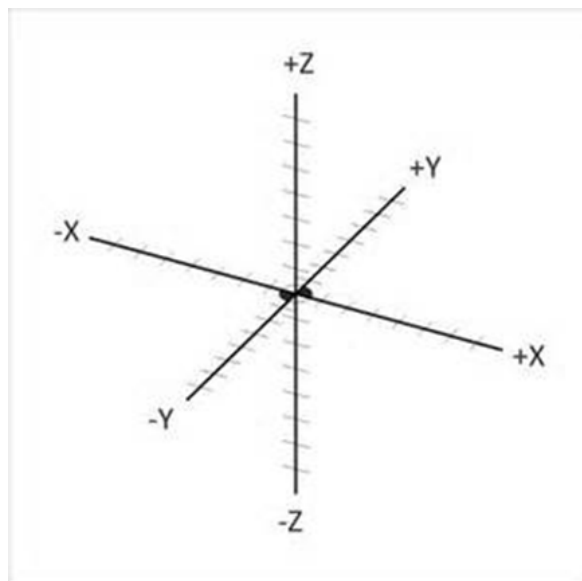
3D Coordinate System



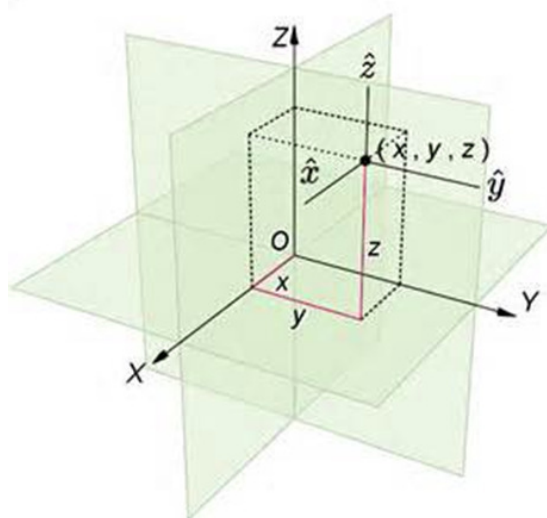
The 3D Coordinate system divide space into 8 octants



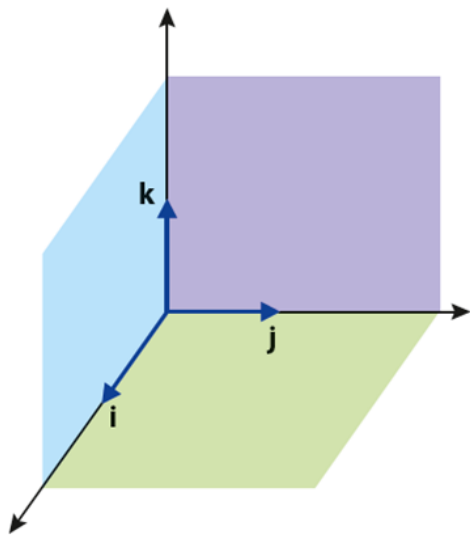
Axes Numbering



Plotting Points in 3D



Base Vectors in 3D



i = unit vector in the direction of the positive x-axis

j = unit vector in the direction of the positive y-axis.

k = unit vector in the direction of the positive z-axis.

$$\vec{v} = \langle x, y, z \rangle = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

The magnitude of a vector in 3D

$$|\vec{v}| = \sqrt{x^2 + y^2 + z^2}$$

Because we now have 3 dimensions, we will have to redefine the meaning of direction (cannot think of it as a slope nor as an angle from the positive x-axis).

Two vectors are parallel if one of them is a scalar multiple of the other.

If $\vec{u} = \langle x, y, z \rangle$ and $\vec{v} = k \langle x, y, z \rangle = \langle kx, ky, kz \rangle$,
then $\vec{u} \parallel \vec{v}$.

Example: \vec{u} , \vec{v} , and \vec{w} are all parallel

$$\vec{u} = \langle 2, 3, -5 \rangle$$

$$\vec{v} = \langle 8, 12, -20 \rangle$$

$$\vec{w} = \left\langle 1, \frac{3}{2}, -\frac{5}{2} \right\rangle$$

Ex1. Given the points A (3, -1, 2) and B (6, 5, -3)

a.) Find the components of vector \overline{AB}

$$\overrightarrow{AB} = B - A = \langle 3, 6, -5 \rangle$$

b.) Find the components of vector \overline{BA}

$$\overrightarrow{BA} = \langle -3, -6, 5 \rangle$$

c.) Find the components of vector $2\overline{AB}$

$$2\overrightarrow{AB} = \langle 6, 12, -10 \rangle$$

d.) Find $|\overline{AB}|$

$$\sqrt{3^2 + 6^2 + (-5)^2} \\ = \sqrt{70}$$

e.) Find $|2\overline{AB}|$

$$\sqrt{6^2 + 12^2 + (-10)^2} = \\ \sqrt{280} = 2\sqrt{70}$$

Ex2. $\vec{v} = 3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$

a.) Find a unit vector in the direction of \vec{v}

$$|\vec{v}| = \sqrt{3^2 + (-2)^2 + 5^2} = \sqrt{38}$$

$$\frac{3}{\sqrt{38}}\mathbf{i} - \frac{2}{\sqrt{38}}\mathbf{j} + \frac{5}{\sqrt{38}}\mathbf{k}$$

b.) Find vector that is 12 units long in the same direction as \vec{v}

$$\frac{36}{\sqrt{38}}\mathbf{i} - \frac{24}{\sqrt{38}}\mathbf{j} + \frac{60}{\sqrt{38}}\mathbf{k}$$

Ex3. Determine the relationship between the coordinates of point M (x, y, z) so that the points M, A (3, 1, 5), and B (-1, 2, -3) are collinear.

$$\begin{aligned} \vec{AB} &\parallel \vec{AM} \\ \langle -4, 1, -8 \rangle &\parallel \langle x-3, y-1, z-5 \rangle \\ k \langle -4, 1, -8 \rangle &= \langle x-3, y-1, z-5 \rangle \\ \boxed{k \langle -4, 1, -8 \rangle + (3, 1, 5) = (x, y, z)} \\ k \in \mathbb{R} \quad k \neq 0, 1 \\ M &= (3, 1, 5) + k \langle -4, 1, -8 \rangle \\ k=2 \quad M &= (3, 1, 5) + \langle -8, 2, -16 \rangle \\ M &= (-5, 3, -11) \end{aligned}$$

pg 634 #1ad, 2ac, 3b, 5ac, 6a, 8b, 9c,
10, 11a, 13 (14, 15, or 16), 17, 19, 20,
23, 25